Math 10B with Professor Stankova
Quiz 10; Tuesday, 4/9/2019
Section #206; Time: 9:30 AM
GSI name: Roy Zhao **Name:** _____

Circle True or False or leave blank. (1 point for correct answer, -1 for incorrect answer, 0 if left blank)

1. True **FALSE** If the mean of a distribution exists, then the standard deviation exists.

Solution: The mean can exist but sometimes the standard deviation doesn't.

2. True **FALSE** Chebyshev's inequality only works for continuous random variables (PDFs).

Solution: It works for PMFs as well.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (7 points) Calculate the standard deviation of $f(x) = \begin{cases} 3x^{-4} & x \leq -1 \\ 0 & x > -1 \end{cases}$ (do not use any formulas).

Solution: First we need to calculate the mean. The mean is

$$\int_{-\infty}^{\infty} xf(x)dx = \int_{-\infty}^{-1} x(3x^{-4})dx = \frac{-3x^{-2}}{2}|_{-\infty}^{-1} = \frac{-3}{2}.$$

Then the variance is

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)dx - \frac{(-3)^2}{2^2} = \int_{-\infty}^{-1} 3x^{-2} - \frac{9}{4}$$

$$= \frac{-3}{x}|_{-\infty}^{-1} - \frac{9}{4} = 3 - \frac{9}{4} = \frac{3}{4}.$$

So the standard deviation is $\frac{\sqrt{3}}{2}$.

(b) (3 points) Let f be a PDF with mean 0 and standard deviation 1. For what value of a can we say that $P(-a \leq X \leq a) \geq 0.99 = \frac{99}{100}$?

Solution: We know that $P(-a \leq X \leq a) = P(\mu - a\sigma \leq X \leq \mu + a\sigma) \geq 1 - \frac{1}{a^2}$. So we need that $0.99 = 1 - \frac{1}{a^2}$ so $a^2 = 100$ and $a = 10$.